Vector Algebra

Question1

A, B, C and D, are any four points. If E and F are mid-points of AC and BD respectively, then $\mathbf{AB}+\mathbf{CB}+\mathbf{CD}+\mathbf{AD}=$

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Options:

A.

EF

B.

$2EF$

C.

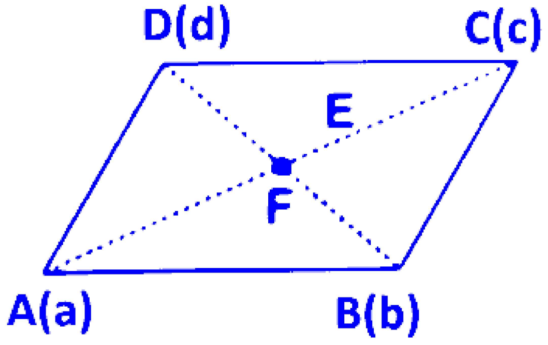
3 EF

D.

$4EF$

Answer: D

Solution:



Let position vector of A, B, C and D be respectively $\mathbf{a}, \mathbf{b}, \mathbf{c}$ and $\mathbf{d}$

$$\therefore \quad AB = b - a, \quad CB = b - c$$
$$CD = d - c, \quad AD = d - a$$
$$\therefore AB + CB + CD + AD$$
$$= 2b + 2d - 2a - 2c$$
$$= 2(b + d - a - c) \quad \ldots (i)$$

And let position vector of E and F are $\mathbf{e}$ and $\mathbf{f}$

$$\therefore \quad \mathbf{e} = \frac{\mathbf{a}+\mathbf{c}}{2}$$
$$\Rightarrow \quad \mathbf{a} + \mathbf{c} = 2\mathbf{e}$$
$$\text{And} \quad \mathbf{f} = \frac{\mathbf{b}+\mathbf{d}}{2} \Rightarrow \mathbf{b} + \mathbf{d} = 2\mathbf{f}$$

$\therefore$ From Eq. (i), we get

$$\mathbf{AB} + \mathbf{CB} + \mathbf{CD} + \mathbf{AD} = 2(2\mathbf{f} - 2\mathbf{c})$$
$$= 4(\mathbf{f} - \mathbf{e}) = 4\mathbf{EF}$$

Question2

The four points whose position vectors are given by $2a + 3b - c, a - 2b + 3c, 3a + 4b - 2c$ and $a - 6b + 6c$ are

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Options:

A.

collinear

B.

coplanar

C.

Vertices of a square

D.

Vertices of a rectangle

Answer: B

Solution:

We have, $\mathbf{OA} = 2\mathbf{a} + 3\mathbf{b} - \mathbf{c}$

$$\begin{aligned} OB &= a - 2b + 3c \\ OC &= 3a + 4b - 2c \\ OD &= a - 6b + 6c \\ \therefore AB &= -a - 5b + 4c \\ AC &= a + b - c \\ AD &= -a - 9b + 7c \end{aligned}$$

$\therefore [\mathbf{AB} \quad \mathbf{AC} \quad \mathbf{AD}]$

$$\begin{aligned} &= \begin{vmatrix} -1 & -5 & 4 \\ 1 & 1 & -1 \\ -1 & -9 & 7 \end{vmatrix} \\ &= (-1)(7-9) + 5(7-1) + 4(-9+1) \\ &= 2 + 30 - 32 = 0 \end{aligned}$$

Given, point are coplanar.

Question3

If $a = |\mathbf{a}|; b = |\mathbf{b}|$, then $\left(\frac{\mathbf{a}}{a^2} - \frac{\mathbf{b}}{b^2}\right)^2$

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Options:

A.

$\left(\frac{a-b}{a^2b^2}\right)^2$

B.

$\left(\frac{\mathbf{a}-\mathbf{b}}{\mathbf{ab}}\right)^{\mathbf{2}}$

C.

$\left(\frac{b\mathbf{a}-a\mathbf{b}}{ab}\right)^2$

D.

$\left(\frac{a\mathbf{a}-b\mathbf{b}}{a^2b^2}\right)^2$

Answer: B

Solution:

Given, $a=|\mathbf{a}|, b=|\mathbf{b}|$

$$\left(\frac{\mathbf{a}}{a^2}-\frac{\mathbf{b}}{b^2}\right)^2=\left(\frac{\mathbf{a}}{a^2}-\frac{\mathbf{b}}{b^2}\right)\left(\frac{\mathbf{a}}{a^2}-\frac{\mathbf{b}}{b^2}\right)$$
$$=\left|\frac{\mathbf{a}}{a^2}\right|^2+\left|\frac{\mathbf{b}}{b^2}\right|^2-2\left(\frac{\mathbf{a}}{a^2}\right)\cdot\left(\frac{\mathbf{b}}{b^2}\right)$$
$$=\frac{|\mathbf{a}|^2}{a^4}+\frac{|\mathbf{b}|^2}{b^4}-\frac{2(\mathbf{a}\cdot\mathbf{b})}{a^2b^2}$$
$$=\frac{1}{a^2}+\frac{1}{b^2}-\frac{2(\mathbf{a}\cdot\mathbf{b})}{a^2b^2}=\frac{a^2+b^2-2(\mathbf{a}\cdot\mathbf{b})}{a^2b^2}$$
$$=\frac{(\mathbf{a}-\mathbf{b})^2}{a^2b^2}=\left(\frac{\mathbf{a}-\mathbf{b}}{ab}\right)^2$$

Question4

a, b, c are three unit vectors such that $x\mathbf{a}+y\mathbf{b}+z\mathbf{c}=p(\mathbf{b}\times\mathbf{c})+q(\mathbf{c}\times\mathbf{a})+r(\mathbf{a}\times\mathbf{b})$. If $(\mathbf{a},\mathbf{b})=(\mathbf{b},\mathbf{c})=(\mathbf{c},\mathbf{a})=\frac{\pi}{3}$, $(\mathbf{a},\mathbf{b}\times\mathbf{c})=\frac{\pi}{6}$ and a, b, c form a right-handed system, then $\frac{x+y+z}{p+q+r}=$

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Options:

A.

$\frac{3}{4}$

B.

$\frac{1}{\sqrt{2}}$

C.

$2\sqrt{2}$

D.

$\frac{3}{8}$

Answer: D

Solution:

We have, $|\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}| = 1$

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \frac{\pi}{3}$$
$$= (1)(1)\left(\frac{1}{2}\right) = \frac{1}{2}$$
$$\mathbf{b} \cdot \mathbf{c} = |\mathbf{b}||\mathbf{c}| \cos \frac{\pi}{3}$$
$$= (1)(1)(\frac{1}{2}) = \frac{1}{2}$$

And $\mathbf{c} \cdot \mathbf{a} = |\mathbf{c}||\mathbf{a}| \cos \frac{\pi}{3} = (1)(1)\left(\frac{1}{2}\right) = \frac{1}{2}$

$$[\mathbf{abc}] = \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$$
$$= |\mathbf{a}||\mathbf{b} \times \mathbf{c}| \cos \frac{\pi}{6}$$

$= (1)\,|\mathbf{b} \times \mathbf{c}|\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2}|\mathbf{b} \times \mathbf{c}|$

Now, $|\mathbf{b} \times \mathbf{c}| = |\mathbf{b}||\mathbf{c}| \sin \frac{\pi}{3}$

$$= (1)(1)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2}$$
$$\therefore \quad [\mathbf{abc}] = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{3}{4}$$

Given, $x\mathbf{a} + y\mathbf{b} + z\mathbf{c} = p(\mathbf{b} \times \mathbf{c}) + q(\mathbf{c} \times \mathbf{a}) + \mathbf{r}(\mathbf{a} \times \mathbf{b})$

$$\therefore \quad x(\mathbf{a} \cdot \mathbf{a}) + y(\mathbf{b} \cdot \mathbf{a}) + z(\mathbf{c} \cdot \mathbf{a})$$
$$= p((\mathbf{b} \times \mathbf{c}) \cdot \mathbf{a})$$
$$\Rightarrow \quad x + y\left(\frac{1}{2}\right) + z\left(\frac{1}{2}\right) = p[\mathbf{abc}]$$
$$\Rightarrow \quad 2x + y + z = \frac{3}{2}p \qquad \ldots(i)$$

Similarly,

$$x(\mathbf{a}\cdot\mathbf{b})+y(\mathbf{b}\cdot\mathbf{b})+z(\mathbf{c}\cdot\mathbf{b})=q((\mathbf{c}\times\mathbf{a})\cdot\mathbf{b})$$
$$\Rightarrow \quad x\left(\frac{1}{2}\right)+y+z\left(\frac{1}{2}\right)=q[\mathbf{abc}]$$
$$\Rightarrow \quad x+2y+z=\frac{3}{2}q \qquad \ldots(ii)$$

And $x(\mathbf{a}\cdot\mathbf{c})+y(\mathbf{b}\cdot\mathbf{c})+z(\mathbf{c}\cdot\mathbf{c})=r[\mathbf{abc}]$

$$\Rightarrow \quad x\left(\frac{1}{2}\right)+y\left(\frac{1}{2}\right)+z=r\left(\frac{3}{4}\right)$$
$$\Rightarrow \quad x+y+2z=\frac{3}{2}r \qquad \ldots(ii)$$

On adding Eqs. (i), (ii) and (iii), we get

$$4(x+y+z)=\frac{3}{2}(p+q+r)$$
$$\Rightarrow \quad \frac{x+y+z}{p+q+r}=\frac{3}{8}$$

Question5

$O(0,0,0)$, $A(3,1,4)$, $B(1,3,2)$ and $C(0,4,-2)$ are the vertices of a tetrahedron. If G is the centroid of the tetrahedron and G_1 is the centroid of its face ABC, then the point which divides GG_1 in the ratio $1:2$ is

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Options:

A.

$\left(\frac{10}{3},\frac{20}{3},\frac{10}{3}\right)$

B.

$\left(\frac{20}{9},\frac{10}{9},\frac{10}{9}\right)$

C.

$\left(\frac{10}{9},\frac{20}{9},\frac{10}{9}\right)$

D.

$\left(\frac{20}{3},\frac{10}{3},\frac{10}{3}\right)$

Answer: C

Solution:

We have, vertices of a tetrahedron $O(0,0,0), A(3,1,4), B(1,3,2)$ and $C(0,4,-2)$

$\therefore$ Centroid of tetrahedron, G

$$= \left(\frac{0+3+1+1+0}{4}, \frac{0+1+3+4}{4}, \frac{0+4+2-2}{4}\right)$$
$$= \left(\frac{4}{4}, \frac{8}{4}, \frac{4}{4}\right) = (1,2,1)$$

and vertices of face ABC

$A(3,1,4), B(1,3,2)$ and $C(0,4,-2)$

$\therefore$ Centroid of face ABC,

$$G_1 = \left(\frac{3+1+0}{3}, \frac{1+3+4}{3}, \frac{4+2-2}{3}\right)$$
$$= \left(\frac{4}{3}, \frac{8}{3}, \frac{4}{3}\right)$$

Let $P(x,y)$ be the point which divides GG_1 in the ratio $1:2$ is

$$\left(\frac{1\times\frac{4}{3}+2\times1}{1+2}, \frac{1\times\frac{8}{3}+2\times2}{1+2}, \frac{1\times\frac{4}{3}+2\times1}{1+2}\right)$$

$$\therefore P(x,y,z) = \left(\frac{4+6}{3(3)}, \frac{8+12}{3(3)}, \frac{4+6}{3(3)}\right)$$
$$\Rightarrow P(x,y,z) = \left(\frac{10}{9}, \frac{20}{9}, \frac{10}{9}\right)$$

Question6

The position vectors of two points A and B are $\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ and $7\hat{\mathbf{i}}-\hat{\mathbf{k}}$ respectively. The point P with position vector $-2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+5\hat{\mathbf{k}}$ is on the line AB. If the point Q is the harmonic conjugate of P, then the sum of the scalar components of the position vector of Q is

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Options:

A.

6

B.

4

C.

2

D.

0

Answer: A

Solution:

Given, $\mathbf{A}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}, \mathbf{B}=7\hat{\mathbf{i}}-\hat{\mathbf{k}}$,

$\mathbf{P}=-2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+5\hat{\mathbf{k}}$

If P divides AB in the ratio $m:n$, then its harmonic conjugate Q divides AB in the ratio $-m:n$.

Now, $\mathbf{AB}=\mathbf{B}-\mathbf{A}$

$$=(7-1)\hat{\mathbf{i}}+(0-2)\hat{\mathbf{j}}+(-1-3)\hat{\mathbf{k}}$$
$$=6\hat{\mathbf{i}}-2\hat{\mathbf{j}}-4\hat{\mathbf{k}}$$

Let P divides AB in the ratio $m:n$, so

$\mathbf{P}=\frac{n\mathbf{A}+m\mathbf{B}}{m+n}$

$$\Rightarrow \quad -2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+5\hat{\mathbf{k}} = \frac{n(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}})+m(7\hat{\mathbf{i}}-\hat{\mathbf{k}})}{m+n}$$
$$\Rightarrow \quad (m+n)(-2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+5\hat{\mathbf{k}}) = n(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}})+m(7\hat{\mathbf{i}}-\hat{\mathbf{k}})$$

Comparing the coefficients of $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ on both sides

$$\Rightarrow \quad -2(m+n)=n(1)+m(7)$$
$$\Rightarrow \quad -2m-2n=n+7m$$
$$\Rightarrow \quad -9m=3n$$
$$\Rightarrow \quad n=-3m$$
$$\Rightarrow \quad \frac{m}{n}=\frac{1}{-3}$$
$$\therefore \quad \frac{-m}{n}=\frac{-1}{-3}=\frac{1}{3}$$

Now using section formula,

$$Q=\frac{3A+1B}{3+1}$$
$$=\frac{3(\hat{i}+2\hat{j}+3\hat{k})+(7\hat{i}-\hat{k})}{4}$$
$$=\frac{10\hat{i}+6\hat{j}+8\hat{k}}{4}=\frac{5}{2}\hat{i}+\frac{3}{2}\hat{j}+2\hat{k}$$

So, sum of scalar components of

$$Q=\frac{5}{2}+\frac{3}{2}+2$$
$$=\frac{8}{2}+2=4+2=6$$

Question7

The point of intersection of the line joining the points $\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}}, 2\hat{\mathbf{i}}-\hat{\mathbf{j}}-\hat{\mathbf{k}}$ and the plane passing through the points $\hat{\mathbf{i}}, 2\hat{\mathbf{j}}, 3\hat{\mathbf{k}}$ is

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Options:

A.

$\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$

B.

$\frac{1}{7}(3\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}})$

C.

$\hat{\mathbf{i}}-3\hat{\mathbf{j}}-2\hat{\mathbf{k}}$

D.

$\frac{1}{7}(15\hat{\mathbf{i}}-10\hat{\mathbf{j}}-9\hat{\mathbf{k}})$

Answer: D

Solution:

Let the points be $\mathbf{A}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $\mathbf{B}=2\hat{\mathbf{i}}-\hat{\mathbf{j}}-\hat{\mathbf{k}}$ and plane through points are $\mathbf{P}_1=\hat{\mathbf{i}}, \mathbf{P}_2=2\hat{\mathbf{j}}, \mathbf{P}_3=3\hat{\mathbf{k}}$

Let the point on the line be

$$\mathbf{r}(t) = \mathbf{A} + t(\mathbf{B} - \mathbf{A})$$

$$\Rightarrow \mathbf{r}(t) = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}) + t[(2\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}})$$
$$(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}})]$$
$$= (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}) + t(\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$$
$$= (1+t)\hat{\mathbf{i}} + (2-3)\hat{\mathbf{j}} + (1-2t)\hat{\mathbf{k}}$$

Now, vectors in the plane are

$$\mathbf{v}_1 = \mathbf{P}_2 - \mathbf{P}_1 = 2\hat{\mathbf{j}} - \hat{\mathbf{i}}$$
$$\mathbf{v}_2 = \mathbf{P}_3 - \mathbf{P}_1 = 3\hat{\mathbf{k}} - \hat{\mathbf{i}}$$

Normal vectors, $\mathbf{n} = \mathbf{v}_1 \times \mathbf{v}_2$

$$\Rightarrow \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -1 & 2 & 0 \\ -1 & 0 & 3 \end{vmatrix} = (6-0)\hat{\mathbf{i}} - (-3-0)\hat{\mathbf{j}} + (0+2\hat{\mathbf{k}}$$

$$\Rightarrow \quad 6\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$$

Using point $\mathbf{P}_1 = \hat{\mathbf{i}}$ to get plane equation is

$$\mathbf{n} \cdot (\mathbf{r} - \hat{\mathbf{i}}) = 0$$

$$\Rightarrow \quad \langle 6, 3, 2 \rangle \cdot \langle x-1, y, z \rangle = 0$$
$$\Rightarrow \quad 6x - 6 + 3y + 2z = 0$$
$$\Rightarrow \quad 6x + 3y + 2z = 6 \qquad \ldots (i)$$

Put the value of $\mathbf{r}(t) = (1+t, 2-3t, 1-2t)$ into the plane Eq. (i), we get

$$6x + 3y + 2z = 6$$
$$\Rightarrow 6(1+t) + 3(2-3t) + 2(1-2t) = 6$$
$$\Rightarrow 6 + 6t + 6 - 9t + 2 - 4t = 6$$
$$\Rightarrow -7t + 14 = 6 \Rightarrow 7t = 8$$
$$\Rightarrow \quad t = \frac{8}{7}$$

$$\text{So, } \mathbf{r}\left(\frac{8}{7}\right) = \left(1 + \frac{8}{7}\right)\hat{\mathbf{i}} + \left(2 - 3 \times \frac{8}{7}\right)\hat{\mathbf{j}} + \left(1 - 2 \times \frac{8}{7}\right)\hat{\mathbf{k}}$$
$$\Rightarrow \frac{15}{7}\hat{\mathbf{i}} - \frac{10}{7}\hat{\mathbf{j}} - \frac{9}{7}\hat{\mathbf{k}}$$

Question8

If a and b are two vectors such that $|\mathbf{a}| = 5, |\mathbf{b}| = 12$ and $|\mathbf{a} - \mathbf{b}| = 13$, then $|2\mathbf{a} + \mathbf{b}| =$

Options:

A.

$2\sqrt{61}$

B.

15

C.

$61\sqrt{2}$

D.

17

Answer: A

Solution:

Given, $|\mathbf{a}| = 5, |\mathbf{b}| = 12|\mathbf{a} - \mathbf{b}| = 13$

Now, $|\mathbf{a} - \mathbf{b}|^2 = 13^2 = 169$

$$\begin{aligned} \Rightarrow \quad & |\mathbf{a}|^2 + |\mathbf{b}|^2 - 2\mathbf{a} \cdot \mathbf{b} = 169 \\ \Rightarrow \quad & 5^2 + 12^2 - 2\mathbf{a} \cdot \mathbf{b} = 169 \\ \Rightarrow \quad & -2\mathbf{a} \cdot \mathbf{b} = 169 - 25 - 144 = 0 \\ \Rightarrow \quad & \mathbf{a} \cdot \mathbf{b} = 0 \end{aligned}$$

So, $\mathbf{a} \perp \mathbf{b}$

Now, $|2\mathbf{a} + \mathbf{b}|^2 = (2\mathbf{a} + \mathbf{b}) \cdot (2\mathbf{a} + \mathbf{b})$

$$\begin{aligned} \Rightarrow \quad & 4\mathbf{a} \cdot \mathbf{a} + 4\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{b} \\ & = 4|\mathbf{a}|^2 + 4\mathbf{a} \cdot \mathbf{b} + |\mathbf{b}|^2 \\ & = 4(5)^2 + 4 \times 0 + (1)^2 \\ & = 100 + 0 + 144 = 244 \\ \therefore \quad & |\mathbf{2a} + \mathbf{b}| = \sqrt{244} = 2\sqrt{61} \end{aligned}$$

Question9

If $\mathbf{a} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ and $\mathbf{b} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ are two vectors, then $(\mathbf{a} + 2\mathbf{b}) \times (3\mathbf{a} - \mathbf{b})$

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Options:

A.

$2\hat{\mathbf{i}}+6\hat{\mathbf{j}}-5\hat{\mathbf{k}}$

B.

$6\hat{\mathbf{i}}-2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$

C.

$14\hat{\mathbf{i}}+7\hat{\mathbf{j}}-5\hat{\mathbf{k}}$

D.

$14\hat{\mathbf{i}}+42\hat{\mathbf{j}}-35\hat{\mathbf{k}}$

Answer: D

Solution:

Given, $\mathbf{a}=\hat{\mathbf{i}}-2\hat{\mathbf{j}}-2\hat{\mathbf{k}}$ and

$\mathbf{b}=2\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}}$

$\mathbf{a}+2\mathbf{b}=(\hat{\mathbf{i}}-2\hat{\mathbf{j}}-2\hat{\mathbf{k}})+2(2\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}})$

$\Rightarrow \quad (1+4)\hat{\mathbf{i}}+(-2+2)\hat{\mathbf{j}}+(-2+4)\hat{\mathbf{k}}$

$\Rightarrow \quad 5\hat{\mathbf{i}}+0\hat{\mathbf{j}}+2\hat{\mathbf{k}}$

$3\mathbf{a}-\mathbf{b}=3(\hat{\mathbf{i}}-2\hat{\mathbf{j}}-2\hat{\mathbf{k}})-(2\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}})$

$\Rightarrow \quad (3-2)\hat{\mathbf{i}}+(-6-1)\hat{\mathbf{j}}+(-6-2)\hat{\mathbf{k}}$

$\Rightarrow \quad \hat{\mathbf{i}}-7\hat{\mathbf{j}}-8\hat{\mathbf{k}}$

$$\text{Now, } (\mathbf{a}+2\mathbf{b})\times(3\mathbf{a}-\mathbf{b})=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 5 & 0 & 2\\ 1 & -7 & -8\end{vmatrix}$$

$\Rightarrow (0-(-14))\hat{\mathbf{i}}-(-40-2)\hat{\mathbf{j}}+(-35-0)\hat{\mathbf{k}}$

$\Rightarrow 14\hat{\mathbf{i}}+42\hat{\mathbf{j}}-35\hat{\mathbf{k}}$

Question10

$2\hat{\mathbf{i}}-3\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $\hat{\mathbf{i}}+2\hat{\mathbf{j}}-3\hat{\mathbf{k}}$ are the position vectores of two points A and B respectively and C divides AB in the ratio $3:2$: If $3\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ is the position of vector of a point D, then the unit vector in the direction of CD is

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Options:

A. $\frac{1}{7\sqrt{2}}(8\hat{\mathbf{i}}-5\hat{\mathbf{j}}-3\hat{\mathbf{k}})$

B. $\frac{1}{\sqrt{266}}(4\hat{\mathbf{i}}-13\hat{\mathbf{j}}+9\hat{\mathbf{k}})$

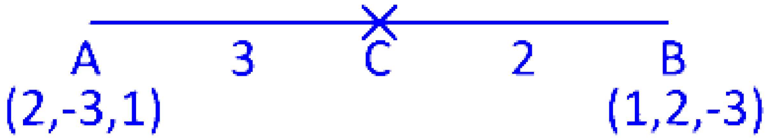
C. $\frac{1}{3\sqrt{42}}(8\hat{\mathbf{i}}-5\hat{\mathbf{j}}+17\hat{\mathbf{k}})$

D. $\frac{1}{7\sqrt{2}}(8\hat{\mathbf{i}}-5\hat{\mathbf{j}}+3\hat{\mathbf{k}})$

Answer: C

Solution:

$C=\left(\frac{344}{5}, \frac{6-6}{5}, \frac{-9+2}{5}\right)$



$C\left(\frac{7}{5}, 0, -\frac{7}{5}\right)$

$OC=\frac{7}{5}\hat{\mathbf{i}}-\frac{7}{5}\hat{\mathbf{k}}$

$OD=3\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}}$

$\mathbf{CD}=\frac{8}{5}\hat{\mathbf{i}}-\hat{\mathbf{j}}+\frac{17}{5}\hat{\mathbf{k}}$

$$\mathbf{CD} = \sqrt{\frac{64}{25} + 1 + \frac{289}{25}} = \sqrt{\frac{378}{25}} = \frac{3\sqrt{42}}{5}$$

$$\mathbf{CD} = \frac{\mathbf{CD}}{|\mathbf{CD}|} = \frac{\frac{1}{5}(8\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 17\hat{\mathbf{k}})}{\frac{3\sqrt{42}}{5}}$$

$$= \frac{1}{3\sqrt{42}}(8\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 17\hat{\mathbf{k}})$$

Question11

A unit vector $\hat{\mathbf{e}} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$ is coplanar with the vectors $\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$, and $3\hat{\mathbf{i}} + \hat{\mathbf{j}} - 5\hat{\mathbf{k}}$. If $\hat{\mathbf{e}}$ is perpendicular to the vector $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, then $2a^2 + 3b^2 + 4c^2 =$

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Options:

A. 1

B. 3

C. -1

D. $\sqrt{2}$

Answer: B

Solution:

To solve for the expression $2a^2 + 3b^2 + 4c^2$, follow these steps:

Given that $\hat{\mathbf{e}} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$ is a unit vector, we have:

$a^2 + b^2 + c^2 = 1$

The vector $\hat{\mathbf{e}}$ is coplanar with the vectors $\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ and $3\hat{\mathbf{i}} + \hat{\mathbf{j}} - 5\hat{\mathbf{k}}$. The condition for coplanarity implies the determinant formed by these vectors should be zero:

$$\begin{vmatrix} a & b & c \\ 1 & -3 & 5 \\ 3 & 1 & -5 \end{vmatrix} = 0$$

Calculating the determinant gives:

$$a(10) - b(-20) + c(10) = 0 \quad \Rightarrow \quad 10a + 20b + 10c = 0$$

Additionally, since $\hat{\mathbf{e}}$ is perpendicular to $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, we have:

$$a + b + c = 0$$

Solving the linear equations, we use the relation derived from coplanarity:

$$10a + 20b + 10c = 0 \quad \Rightarrow \quad a + 2b + c = 0$$

Together with $a + b + c = 0$, it follows:

$$\frac{a}{1} = \frac{-b}{0} = \frac{c}{-1} = \lambda$$

Thus:

$$a = \lambda, \quad b = 0, \quad c = -\lambda$$

Substituting into the unit vector condition:

$$a^2 + b^2 + c^2 = 1 \quad \Rightarrow \quad \lambda^2 + 0 + (-\lambda)^2 = 1 \quad \Rightarrow \quad 2\lambda^2 = 1$$

Hence:

$$\lambda^2 = \frac{1}{2} \quad \Rightarrow \quad \lambda = \pm\frac{1}{\sqrt{2}}$$

Therefore, the values for a, b, and c are:

$$a = \frac{1}{\sqrt{2}}, \quad b = 0, \quad c = -\frac{1}{\sqrt{2}}$$

Calculating $2a^2 + 3b^2 + 4c^2$:

$$2\left(\frac{1}{2}\right) + 3(0) + 4\left(\frac{1}{2}\right) = 1 + 2 = 3$$

Thus, $2a^2 + 3b^2 + 4c^2 = 3$.

Question12

$\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}, \hat{\mathbf{b}} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\hat{\mathbf{c}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ are three vectors. If $\hat{\mathbf{d}}$ is a normal to the plane of $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ and $\mathbf{d}.\,\hat{\mathbf{c}} = 2$, then $|\hat{\mathbf{d}}| =$

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Options:

A. $\sqrt{6}$

B. $2\sqrt{3}$

C. $\sqrt{3}$

D. 2

Answer: C

Solution:

To find the magnitude of vector $\hat{\mathbf{d}}$, which is normal to the plane formed by vectors $\mathbf{a}$ and $\mathbf{b}$, and also satisfies $\hat{\mathbf{d}} \cdot \hat{\mathbf{c}} = 2$, we perform the following calculations:

Firstly, calculate $\mathbf{a} \times \mathbf{b}$, the cross product of vectors $\mathbf{a}$ and $\mathbf{b}$:

$$\mathbf{d} = \lambda(\mathbf{a} \times \mathbf{b}) = \lambda \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & -2 \\ 1 & -2 & 1 \end{vmatrix}$$

The calculation proceeds as follows:

$$\begin{aligned} \mathbf{d} &= \lambda\left(\hat{\mathbf{i}}(1 \cdot 1 - (-2) \cdot (-2)) - \hat{\mathbf{j}}(1 \cdot 1 - (-2) \cdot 1) + \hat{\mathbf{k}}(1 \cdot (-2) - 1 \cdot 1)\right) \\ &= \lambda(\hat{\mathbf{i}}(1-4) - \hat{\mathbf{j}}(1+2) + \hat{\mathbf{k}}(-2-1)) \\ &= \lambda(-3\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) \\ &= -3\lambda(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) \end{aligned}$$

Next, use the condition $\hat{\mathbf{d}} \cdot \hat{\mathbf{c}} = 2$:

$$\mathbf{d} \cdot \mathbf{c} = -3\lambda[2\hat{\mathbf{i}} + 1\hat{\mathbf{j}} - 1\hat{\mathbf{k}}]$$

Solves as:

$$\begin{aligned} -3\lambda[2+1-1] &= 2 \\ -3\lambda \times 2 &= 2 \\ -6\lambda &= 2 \\ \lambda &= -\frac{1}{3} \end{aligned}$$

Substitute λ back to determine $\mathbf{d}$:

$$\mathbf{d} = \lambda(-3(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})) = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$$

Finally, the magnitude of $\hat{\mathbf{d}}$ is:

$$|\mathbf{d}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

Question13

If $\mathbf{a} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}, \mathbf{c} = -\hat{\mathbf{k}}$ are position vectors of two points and $\mathbf{b} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \lambda\hat{\mathbf{k}}, \mathbf{d} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ are two vectors, then the lines $\mathbf{r} = \mathbf{a} + t\mathbf{b}, \mathbf{r} = \mathbf{c} + s\mathbf{d}$ are

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Options:

A. skew lines, when $\lambda = \frac{19}{3}$

B. coplanar, $\forall \lambda \in R$

C. skew lines when $\lambda \neq \frac{19}{3}$

D. coplanar, when $\lambda \neq \frac{19}{3}$

Answer: C

Solution:

The position vectors and direction vectors for two lines are given:

Line L_1: $\mathbf{r} = \mathbf{a} + t\mathbf{b}$, where $\mathbf{a} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\mathbf{b} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \lambda\hat{\mathbf{k}}$.

Line L_2: $\mathbf{r} = \mathbf{c} + s\mathbf{d}$, where $\mathbf{c} = -\hat{\mathbf{k}}$ and $\mathbf{d} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$.

To determine whether the lines are coplanar or skew, we set up equations based on their parametric forms:

$$L_1 = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} + t \begin{pmatrix} 2 \\ -1 \\ \lambda \end{pmatrix}$$

$$L_2 = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} + s \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

By equating the components of the parametric equations, we get:

$1 + 2t = s$

$-1 - t = 2s$

$3 + \lambda t = -1 - s$

Solving the first two equations simultaneously gives:

$2 + 4t = 2s$

$-1 - t = 2s$

$\frac{-1-t}{2s} = \frac{3+5t}{0}$

From these equations, solve for t and s:

$\Rightarrow t = -\frac{3}{5}$

$\Rightarrow s = -\frac{1}{5}$ after substituting t.

For the third equation:

$3 - \frac{3\lambda}{5} = -1 + \frac{1}{5}$

From this, solve for λ:

$21 - \frac{1}{5} = \frac{3\lambda}{5}$

$\Rightarrow \lambda = \frac{19}{3}$

Thus, the lines are skew when $\lambda \neq \frac{19}{3}$. If $\lambda = \frac{19}{3}$, the lines would be coplanar.

Question14

a, b, c are three vectors each having $\sqrt{2}$ magnitude such that $(\mathbf{a}, \mathbf{b}) = (\mathbf{b}, \mathbf{c}) = (\mathbf{c}, \mathbf{a}) = \frac{\pi}{3}$. If $\mathbf{x} = \mathbf{a} \times (\mathbf{b} \times \mathbf{c})$ and $\mathbf{y} = \mathbf{b} \times (\mathbf{c} \times \mathbf{a})$, then

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Options:

A. $|\mathbf{x}| = |y|$

B. $|x| = \sqrt{2}|y|$

C. $|x| = 2|y|$

D. $|x| + |y| = 2$

Answer: A

Solution:

Given that the vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ each have a magnitude of $\sqrt{2}$ and the angle between any two vectors is $\frac{\pi}{3}$, we can analyze the vectors $\mathbf{x}$ and $\mathbf{y}$ defined as follows:

$|\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}| = \sqrt{2}$

First, let's express $\mathbf{x}$:

$\mathbf{x} = \mathbf{a} \times (\mathbf{b} \times \mathbf{c})$

Using the vector triple product identity:

$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$

Given that the angle between each pair of vectors is $\frac{\pi}{3}$, the dot products are:

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\left(\frac{\pi}{3}\right) = \sqrt{2} \times \sqrt{2} \times \frac{1}{2} = 1$

Therefore, the expression for $\mathbf{x}$ becomes:

$\mathbf{x} = 1 \cdot \mathbf{b} - 1 \cdot \mathbf{c} = \mathbf{b} - \mathbf{c}$

Now, consider $\mathbf{y}$:

$\mathbf{y} = \mathbf{b} \times (\mathbf{c} \times \mathbf{a})$

Using the vector triple product identity again:

$\mathbf{b} \times (\mathbf{c} \times \mathbf{a}) = (\mathbf{b} \cdot \mathbf{a})\mathbf{c} - (\mathbf{b} \cdot \mathbf{c})\mathbf{a}$

This simplifies similarly to:

$\mathbf{y} = 1 \cdot \mathbf{c} - 1 \cdot \mathbf{a} = \mathbf{c} - \mathbf{a}$

Hence, the magnitudes of $\mathbf{x}$ and $\mathbf{y}$ are:

$|\mathbf{x}| = |\mathbf{b} - \mathbf{c}|$

$|\mathbf{y}| = |\mathbf{c} - \mathbf{a}|$

Since both expressions have the same form, it follows that:

$|\mathbf{x}| = |\mathbf{y}|$

Question15

a is a vector perpendicular to the plane containing non zero vectors b and c. If a, b, c are such that

$|\mathbf{a} + \mathbf{b} + \mathbf{c}| = \sqrt{|\mathbf{a}|^2 + |\mathbf{b}|^2 + |\mathbf{c}|^2}$**, then**

$|(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}| + |(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}| =$

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Options:

A. $|\mathbf{a}|+|\mathbf{b}|+|\mathbf{c}|$

B. $|\mathbf{a}||\mathbf{b}||\mathbf{c}|$

C. $|a|^2+|b|^2+|\ d^2$

D. $|\mathbf{a}|^2|\mathbf{b}|^2|\mathbf{c}|^2$

Answer: B

Solution:

Given that the vector $\mathbf{a}$ is perpendicular to the plane formed by the non-zero vectors $\mathbf{b}$ and $\mathbf{c}$, we have:

$\mathbf{a} \cdot \mathbf{b}=0$

$\mathbf{a} \cdot \mathbf{c}=0$

From the problem statement, we know:

$|\mathbf{a}+\mathbf{b}+\mathbf{c}|=\sqrt{|\mathbf{a}|^2+|\mathbf{b}|^2+|\mathbf{c}|^2}$

Expanding both sides, we get:

$|\mathbf{a}|^2+|\mathbf{b}|^2+|\mathbf{c}|^2+2(\mathbf{a} \cdot \mathbf{b}+\mathbf{b} \cdot \mathbf{c}+\mathbf{c} \cdot \mathbf{a})=|\mathbf{a}|^2+|\mathbf{b}|^2+|\mathbf{c}|^2$

This simplifies to:

$2(\mathbf{a} \cdot \mathbf{b}+\mathbf{b} \cdot \mathbf{c}+\mathbf{c} \cdot \mathbf{a})=0$

Hence, we conclude:

$\mathbf{a} \cdot \mathbf{b}+\mathbf{b} \cdot \mathbf{c}+\mathbf{c} \cdot \mathbf{a}=0$

Now, considering $|(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}|+|(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}|$, we find:

$|(|\mathbf{a}||\mathbf{b}| \sin 90^{\circ}) \cdot \mathbf{c}|+|(\mathbf{a} \cdot \mathbf{c}) \mathbf{b}-(\mathbf{b} \cdot \mathbf{c}) \mathbf{a}|$

Since $\sin 90^{\circ}=1$, this becomes:

$|\mathbf{a}||\mathbf{b}||\mathbf{c}|+0-0 \quad$ (because $\mathbf{a} \cdot \mathbf{c}=0$ and $\mathbf{b} \cdot \mathbf{c}=0$)

Thus, the result is:

$|\mathbf{a}||\mathbf{b}||\mathbf{c}|$

Question16

If $\mathbf{a}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}}, \mathbf{b}=3(\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}})$ and $\mathbf{c}$ is a vector such that $\mathbf{a} \times \mathbf{c}=\mathbf{b}$ and $\mathbf{a} . \mathbf{c}=3$, then $\mathbf{a} \cdot(\mathbf{c} \times \mathbf{b}-\mathbf{b}-\mathbf{c})=$

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Options:

A. 32

B. 24

C. 20

D. 36

Answer: B

Solution:

Given the relationships $\mathbf{a} \times \mathbf{c} = \mathbf{b}$ and $\mathbf{a} \cdot \mathbf{c} = 3$, we need to find $\mathbf{a} \cdot (\mathbf{c} \times \mathbf{b} - \mathbf{b} - \mathbf{c})$.

First, consider:

$(\mathbf{a} \times \mathbf{c})^2 + (\mathbf{a} \cdot \mathbf{c})^2 = |\mathbf{a}|^2 \cdot |\mathbf{c}|^2$

We are given $|\mathbf{b}|^2 = (\mathbf{a} \times \mathbf{c})^2$. Thus:

$|\mathbf{b}|^2 + 9 = |\mathbf{a}|^2 \cdot |\mathbf{c}|^2$

Start with:

$27 + 9 = 6 \cdot |\mathbf{c}|^2 \implies 36 = 6 \cdot |\mathbf{c}|^2 \implies |\mathbf{c}|^2 = 6$

We then know:

$|c|^2\mathbf{a} - (\mathbf{a} \cdot \mathbf{c})\mathbf{c} = \mathbf{c} \times \mathbf{b}$

Substituting:

$6\mathbf{a} - 3\mathbf{c} = \mathbf{c} \times \mathbf{b}$

Now compute:

$\mathbf{a} \cdot (6\mathbf{a} - 3\mathbf{c} - \mathbf{b} - \mathbf{c})$

Breaking it down:

$6\mathbf{a}^2 = 6 \times |\mathbf{a}|^2 = 6 \times 6$

$-4(\mathbf{a} \cdot \mathbf{c}) = -4 \times 3 = -12$

$\mathbf{a} \cdot \mathbf{b} = 0$ because $\mathbf{a} \times \mathbf{c} = \mathbf{b}$ implies orthogonality of $\mathbf{a}$ and $\mathbf{b}$.

Substituting:

$= 6 \times 6 - 12 - 0 = 36 - 12 - 0 = 24$

Thus, the final result is:

24

Question17

P **and** Q **are the points of trisection of the segment** AB**. If** $2\hat{\mathbf{i}}-5\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ **and** $4\hat{\mathbf{i}}+\hat{\mathbf{j}}-6\hat{\mathbf{k}}$ **are the position vectors of** A **and** B **respectively, then the position vector of the point which divides** PQ **in the ratio** $2:3$ **is**

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Options:

A. $\frac{1}{15}(44\hat{\mathbf{i}}-33\hat{\mathbf{j}}-18\hat{\mathbf{k}})$

B. $\frac{1}{5}(36\hat{\mathbf{i}}-26\hat{\mathbf{j}}-18\hat{\mathbf{k}})$

C. $\frac{1}{5}(3\hat{\mathbf{i}}+7\hat{\mathbf{j}}-9\hat{\mathbf{k}})$

D. $\frac{1}{15}(-3\hat{\mathbf{i}}-7\hat{\mathbf{j}}+9\hat{\mathbf{k}})$

Answer: A

Solution:

Position vector of

$$P=\frac{4\hat{\mathbf{i}}+\hat{\mathbf{j}}-6\hat{\mathbf{k}}+4\hat{\mathbf{i}}-10\hat{\mathbf{j}}+6\hat{\mathbf{k}}}{3}=\frac{8\hat{\mathbf{i}}-9\hat{\mathbf{j}}}{3}$$

Position vector of

$$Q=\frac{8\hat{\mathbf{i}}+2\hat{\mathbf{j}}-12\hat{\mathbf{k}}+2\hat{\mathbf{i}}-5\hat{\mathbf{j}}+3\hat{\mathbf{k}}}{3}$$

$$=\frac{10\hat{\mathbf{i}}-3\hat{\mathbf{j}}-9\hat{\mathbf{k}}}{3}$$

Let the point which divides PQ in ratio $2:3$ be M.

Position vector of M

$$=\frac{2\left(\frac{10\hat{\mathbf{i}}-3\hat{\mathbf{j}}-9\hat{\mathbf{k}}}{3}\right)+3\left(\frac{8\hat{\mathbf{i}}-9\hat{\mathbf{j}}}{3}\right)}{5}$$

$= \frac{20\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - 18\hat{\mathbf{k}} + 24\hat{\mathbf{i}} - 27\hat{\mathbf{j}}}{15}$

$= \frac{44\hat{\mathbf{i}} - 33\hat{\mathbf{j}} - 18\hat{\mathbf{k}}}{15}$

Question18

The position vector of the point of intersection of the line joining the points $\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}, \hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ and the line joining the points $2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 6\hat{\mathbf{k}}, 3\hat{\mathbf{i}} - \hat{\mathbf{j}} - 7\hat{\mathbf{k}}$ is

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Options:

A. $\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

B. $4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 8\hat{\mathbf{k}}$

C. $\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$

D. $\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

Answer: C

Solution:

Line joining the points $(1, -1, 1)$ and $(1, 1, -2)$ be

$\frac{x-1}{0} = \frac{y+1}{2} = \frac{z-1}{-3}$

Line joining the points $(2, 1, -6)$ and $(3, -1, -7)$ is

$\frac{x-2}{1} = \frac{y-1}{-2} = \frac{z+6}{-1}$

For intersection of points,

$\frac{x-1}{0} = \frac{y+1}{2} = \frac{z-1}{-3} = K$

$\Rightarrow x = 1, y = 2K - 1$ and $Z = -3K + 1$

and $\frac{x-2}{1} = \frac{y-1}{-2} = \frac{z+6}{-1} = \lambda$

$x=\lambda+2, y=1-2\lambda, z=-\lambda-6$

$\therefore \lambda=-1$ and $K=2$

$\therefore$ Positivion vector of points of intersection is $\mathbf{i}+3\hat{\mathbf{j}}-5\hat{\mathbf{k}}$

Question19

If $\mathbf{a}=4\hat{\mathbf{i}}+5\hat{\mathbf{j}}-3\hat{\mathbf{k}}$ and $\mathbf{b}=6\hat{\mathbf{i}}-2\hat{\mathbf{j}}-2\hat{\mathbf{k}}$ are two vectors, then the magnitude of the component of b parallel to a is

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Options:

A. $2\sqrt{2}$

B. $10\sqrt{2}$

C. $4\sqrt{2}$

D. $6\sqrt{2}$

Answer: A

Solution:

$\mathbf{a}=4\hat{\mathbf{i}}+5\hat{\mathbf{j}}-3\hat{\mathbf{k}}$ and $\mathbf{b}=6\hat{\mathbf{i}}-2\hat{\mathbf{j}}-2\hat{\mathbf{k}}$

Required magnitude $=\frac{\mathbf{a}\cdot\mathbf{b}}{|a|}$

$$=\frac{24-10+6}{\sqrt{16+25+9}}=\frac{20}{\sqrt{50}}=\frac{20}{5\sqrt{2}}$$

$$=2\sqrt{2}$$

Question20

$\mathbf{a}=2\hat{\mathbf{i}}-\hat{\mathbf{j}}, \mathbf{b}=2\hat{\mathbf{j}}-\hat{\mathbf{k}}$ and $\mathbf{c}=2\hat{\mathbf{k}}-\hat{\mathbf{i}}$ are three vectors and d is a unit vector perpendicular to c. If a, b and d are coplanar vectors, then $|\mathbf{d}\cdot\mathbf{b}|=$

Options:

A. 0

B. $\frac{1}{\sqrt{14}}$

C. $\sqrt{\frac{2}{7}}$

D. $\sqrt{\frac{7}{2}}$

Answer: D

Solution:

Let $\mathbf{d} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$

Given, $|\mathbf{d}| = 1 \Rightarrow x^2 + y^2 + z^2 = 1 \quad \ldots$ (i)

$d \cdot \mathbf{c} = 0 \Rightarrow -x + 2z = 0 \Rightarrow x = 2z \quad \ldots$ (ii)

$\mathbf{a}, \mathbf{b}$ and $\mathbf{d}$ are coplanar

$$\begin{vmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ x & y & z \end{vmatrix} = 0$$

$2(2z + y) + x(l) = 0$

$4z + 2y + 2z = 0$

$6z + 2y = 0 \Rightarrow y = -3z$

$[\because x = 2z]$

From Eq. (i), $z^2 + 4z^2 + 9z^2 = 1$

$14z^2 = 1$

$$z^2 = \frac{1}{14} \Rightarrow z = \frac{\pm 1}{\sqrt{14}}$$

If $z = \frac{1}{\sqrt{14}}, x = \frac{2}{\sqrt{14}}, y = \frac{-3}{\sqrt{14}}$

If $z = \frac{-1}{\sqrt{14}}, x = \frac{-2}{\sqrt{14}}, y = \frac{3}{\sqrt{14}}$

$|d \cdot b| = \left(\frac{6}{\sqrt{14}} + \frac{1}{\sqrt{14}}\right) = \frac{7}{\sqrt{14}} = \sqrt{\frac{7}{2}}$

Question21

a, b, c are non-coplanar vectors. If the three points $\lambda a - 2b + c, 2a + \lambda b - 2\mathbf{c}$ and $4\mathbf{a} + 7\mathbf{b} - 8\mathbf{c}$ are collinear, then $\lambda =$

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Options:

A. -1

B. 2

C. -2

D. 1

Answer: D

Solution:

Given,

$\mathbf{a}, \mathbf{b}, \mathbf{c}$ are non-coplanar vectors and the three points $\lambda\mathbf{a} - 2\mathbf{b} + \mathbf{c}, 2\mathbf{a} + \lambda\mathbf{b} - 2\mathbf{c}, 4a + 7b - 8c$ are collinear.

Let $P = \lambda\mathbf{a} - 2\mathbf{b} + \mathbf{c}$

$Q = 2\mathbf{a} + \lambda\mathbf{b} - 2\mathbf{c}$

$R = 4\mathbf{a} + 7\mathbf{b} - 8\mathbf{c}$

$\mathbf{PQ} = (2\mathbf{a} + \lambda\mathbf{b} - 2\mathbf{c}) - (\lambda\mathbf{a} - 2\mathbf{b} + \mathbf{c})$

$= (2 - \lambda)\mathbf{a} + (\lambda + 2)\mathbf{b} + (-3)\mathbf{c}$

$\mathbf{PR} = (4a + 7b - 8\mathbf{c}) - (\lambda\mathbf{a} - 2\mathbf{b} + \mathbf{c})$

$= (4 - \lambda)\mathbf{a} + 9\mathbf{b} - 9\mathbf{c}$

$\because P, Q$ and R collinear.

$\therefore \mathbf{PQ} = k\mathbf{PR}$

$\Rightarrow (2-\lambda)\mathbf{a} + (\lambda+2)\mathbf{b} + (-3)\mathbf{c}$

$= k(4-\lambda)\mathbf{a} + 9k\mathbf{b} - 9k\mathbf{c}$

$\therefore \quad 2-\lambda = k(4-\lambda) \Rightarrow \lambda + 2 = 9k$

$-3 = -9k \Rightarrow k = \frac{1}{3}$

Now, $\lambda + 2 = 9\left(\frac{1}{3}\right) = 3$

$\Rightarrow \quad 6 - 3\lambda \quad = 4 - \lambda \Rightarrow 2 = 2\lambda$

$\Rightarrow \quad \lambda = 1$

Question22

If a, b are two vectors such that $|\mathrm{a}| = 3, |\mathrm{~b}| = 4$, $|\mathbf{a}+\mathbf{b}| = \sqrt{37}, |\mathbf{a}-\mathbf{b}| = k$ and $(\mathbf{a}, \mathbf{b}) = \theta$, then $\frac{4}{13}(k \sin \theta)^2 =$

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Options:

A. 1

B. 2

C. 3

D. 4

Answer: C

Solution:

We have,

$|\mathbf{a}| = 3, |\mathbf{b}| = 4$

$|\mathbf{a}+\mathbf{b}| = \sqrt{37}, |\mathbf{a}-\mathbf{b}| = k$ and

$(\mathbf{a}, \mathbf{b}) = 0$

Now, $|\mathbf{a}+\mathbf{b}| = \sqrt{37}$

$\Rightarrow \quad (\mathbf{a}+\mathbf{b}) \cdot (\mathbf{a}+\mathbf{b}) = 37$

$\Rightarrow \quad |\mathbf{a}|^2 + |\mathbf{b}|^2 + 2\mathbf{a} \cdot \mathbf{b} = 37$

$\Rightarrow \quad 9 + 16 + 2|\mathbf{a}||\mathbf{b}| \cos\theta = 37$

$\Rightarrow \quad 2(3)(4) \cos\theta = 12$

$\Rightarrow \quad \cos\theta = \frac{1}{2}$

$\Rightarrow \quad \theta = 60^\circ$

Now, $|\mathbf{a}-\mathbf{b}| = k$

$\Rightarrow (\mathbf{a}-\mathbf{b}) \cdot (\mathbf{a}-\mathbf{b}) = k^2$

$\Rightarrow |\mathbf{a}|^2 + |\mathbf{b}|^2 - 2|\mathbf{a}||\mathbf{b}| \cos(\theta) = k^2$

$\Rightarrow 9 + 16 - 2(3)(4) \cos(60^\circ) = k^2$

$\Rightarrow 25 - 24\left(\frac{1}{2}\right) = k^2$

$\Rightarrow k^2 = 13$

$\therefore \frac{4}{13}(k \sin\theta)^2$

$= \frac{4}{13} k^2 \sin^2(60^\circ)$

$= \frac{4}{13}(13)\left(\frac{\sqrt{3}}{2}\right)^2 = 3$

Question23

r is a vector perpendicular to the planet, determined by the vectors $2\hat{\mathrm{i}} - \hat{\mathrm{j}}$ and $\hat{\mathrm{j}} + 2\hat{\mathrm{k}}$, If the magnitude of the projection of r on the vector $2\hat{\mathrm{i}} + \hat{\mathrm{j}} + 2\hat{\mathrm{k}}$ is l , then $|\mathbf{r}| =$

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Options:

A. $\sqrt{6}$

B. $3\sqrt{6}$

C. $\frac{2\sqrt{6}}{3}$

D. $\frac{3\sqrt{6}}{2}$

Answer: D

Solution:

Given,

$\mathbf{r}$ is a vector perpendicular to the plane determined by the vectors $2\hat{\mathbf{i}}-\hat{\mathbf{j}}$ and $\hat{\mathbf{j}}+2\hat{\mathbf{k}}$

$$\therefore \quad \mathbf{r}=\lambda\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 2 & -1 & 0\\ 0 & 1 & 2\end{vmatrix}=\lambda(-2\hat{\mathbf{i}}-4\hat{\mathbf{j}}+2\hat{\mathbf{k}})$$

Also, given that

The magnitude of the projection of $\mathbf{r}$ on the vector $2\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ is 1

$$\therefore \quad \left|\frac{\mathbf{r}\cdot(2\hat{i}+\hat{\mathbf{j}}+2\hat{\mathbf{k}})}{\sqrt{2^2+1^2+2^2}}\right|=1$$

$$\Rightarrow \quad |\lambda(-4-4+4)|=3\Rightarrow\lambda=\frac{3}{4}$$

$$\therefore \quad \mathbf{r}=\frac{3}{4}(-2\hat{\mathbf{i}}-4\hat{\mathbf{j}}+2\hat{\mathbf{k}})$$

Now,

$$|r|=\frac{3}{4}\sqrt{(-2)^2+(4)^2+(2)^2}=\frac{3}{4}\sqrt{24}$$

$$=\frac{3}{4}(2\sqrt{6})=\frac{3\sqrt{6}}{2}$$

Question24

$\mathbf{b}=\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\mathbf{k},\quad \mathbf{c}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}$ are two vectors and a is a vector such that$\cos(\mathbf{a},\mathbf{b}\times\mathbf{c})=\sqrt{\frac{2}{3}}$. If a is a unit vector, then $|\mathbf{a}\times(\mathbf{b}\times\mathbf{c})|=$

Options:

A. 3

B. 2

C. 1

D. 4

Answer: A

Solution:

Given,

$\mathbf{b}=\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}}, \mathbf{c}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}$

$\cos(\mathbf{a}, \mathbf{b}\times\mathbf{c})=\sqrt{\frac{2}{3}}$ and $\mathbf{a}$ is a unit vectors.

$$\text{Now, } \mathbf{b}\times\mathbf{c}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 1 & -1 & 2\\ 1 & 2 & -1\end{vmatrix}=-3\hat{\mathbf{i}}+3\hat{\mathbf{j}}+3\hat{\mathbf{k}}$$

$\therefore|\mathbf{a}\times(\mathbf{b}\times\mathbf{c})|$

$=|\mathbf{a}||\mathbf{b}\times\mathbf{c}|\sin(\mathbf{a}, \mathbf{b}\times\mathbf{c})$

$=(1)\sqrt{(-3)^2+(3)^2+(3)^2}\sqrt{1-(\cos(\mathbf{a}, \mathbf{b}\times\mathbf{c}))^2}$

$$=3\sqrt{3}\sqrt{1-\left(\sqrt{\frac{2}{3}}\right)^2}=3\sqrt{3}\left(\frac{1}{\sqrt{3}}\right)=3$$

Question25

$A(3,2,-1), B(4,1,0), C(2,1,4)$ are the vertices of a $\triangle ABC$. If the bisector of BAC ! intersects the side BC at $D(p,q,r)$, then

$\sqrt{2p+q+r} =$

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Options:

A. 3

B. 4

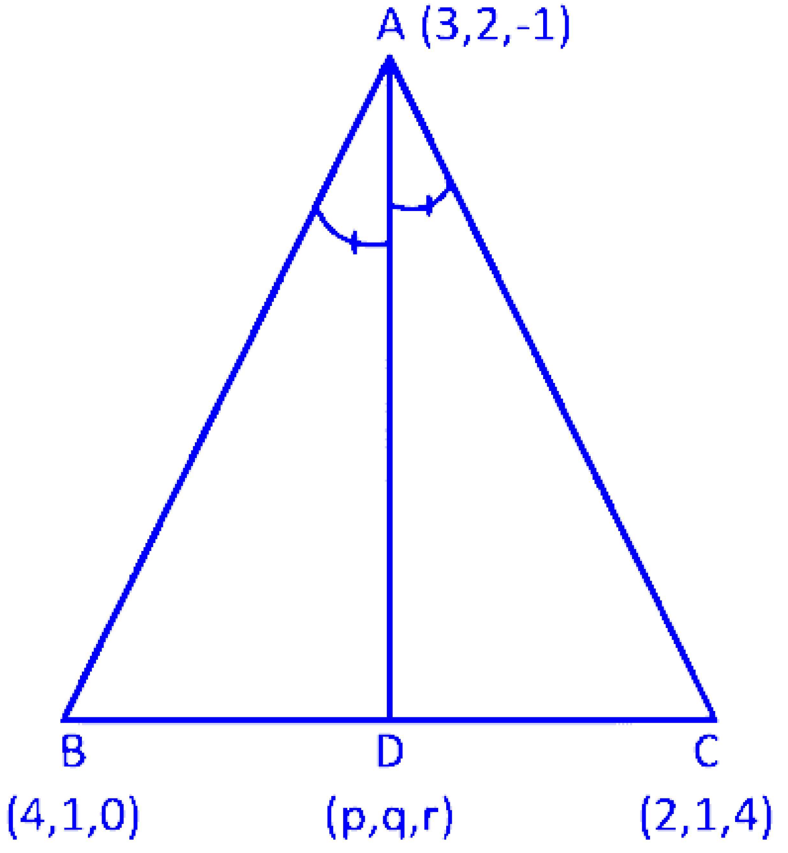
C. 1

D. 2

Answer: C

Solution:

$AB = \sqrt{(4-3)^2 + (1-2)^2 + (0+1)^2}$



$$= \sqrt{1+1+1} = \sqrt{3}$$

$$AC = \sqrt{(2-3)^2 + (1-2)^2 + (4+1)^2}$$

$$= \sqrt{1+1+25}$$

$$= \sqrt{27} = 3\sqrt{3}$$

By angle bisector theorem,

$\frac{BD}{CD} = \frac{AB}{AC} = \frac{\sqrt{3}}{3\sqrt{3}} = \frac{1}{3} = 1:3$

The point D divides the segment BC internally in the ratio $1:3$

$\therefore$ Coordinate of D will be

$\left(\frac{4(3)+2(1)}{1+3}, \frac{1(3)+1(1)}{1+3}, \frac{0(3)+4(1)}{1+3}\right)$

i.e. $\left(\frac{7}{2}, 1, 1\right)$

Given, $D \equiv (p, q, r)$

$\therefore \quad p = \frac{7}{2}, q = 1$ and $r = 1$

Now, $\sqrt{2p+q+r} = \sqrt{2\left(\frac{7}{2}\right)+1+1} = \sqrt{9} = 3$

Question26

$(3, 0, 2)$ and $(0, 2, k)$ are the direction ratios of two lines and θ is the angle between them. If $|\cos\theta| = \frac{6}{13}$, then $k =$

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Options:

A. ± 2

B. ± 3

C. ± 5

D. ± 7

Answer: B

Solution:

Given, $(3, 0, 2)$ and $(0, 2, k)$ are the direction ratios of two lines and θ is the angle between them.

$$\therefore \cos\theta = \frac{3\times 0 + 0\times 2 + 2\times k}{\sqrt{3^2+0^2+2^2}\cdot\sqrt{0^2+2^2+k^2}}$$

$$\Rightarrow |\cos\theta| = \left|\frac{2k}{\sqrt{13\,(4+k^2)}}\right|$$

$$\Rightarrow \left|\frac{2k}{\sqrt{13\,(4+k^2)}}\right| = \frac{6}{13}\left[\text{given, } |\cos\theta| = \frac{6}{13}\right]$$

$$\Rightarrow \quad \frac{4(k)^2}{13\,(4+k^2)} = \left(\frac{6}{13}\right)^2 \Rightarrow \frac{4k^2}{4+k^2} = \frac{36}{13}$$

$$\Rightarrow \quad 52k^2 = 144 + 36k^2$$

$$\Rightarrow \quad 16k^2 = 144 \Rightarrow k^2 = 9$$

$$\Rightarrow \quad k = \pm 3$$

Question27

$\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$, $2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ are the position vectors of the vertices A, B and C of a $\triangle ABC$ respectively. If D and E are the mid points of BC and CA respectively, then the unit vector along DE is

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Options:

A. $\frac{1}{7}(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}})$

B. $\frac{1}{\sqrt{14}}(-\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

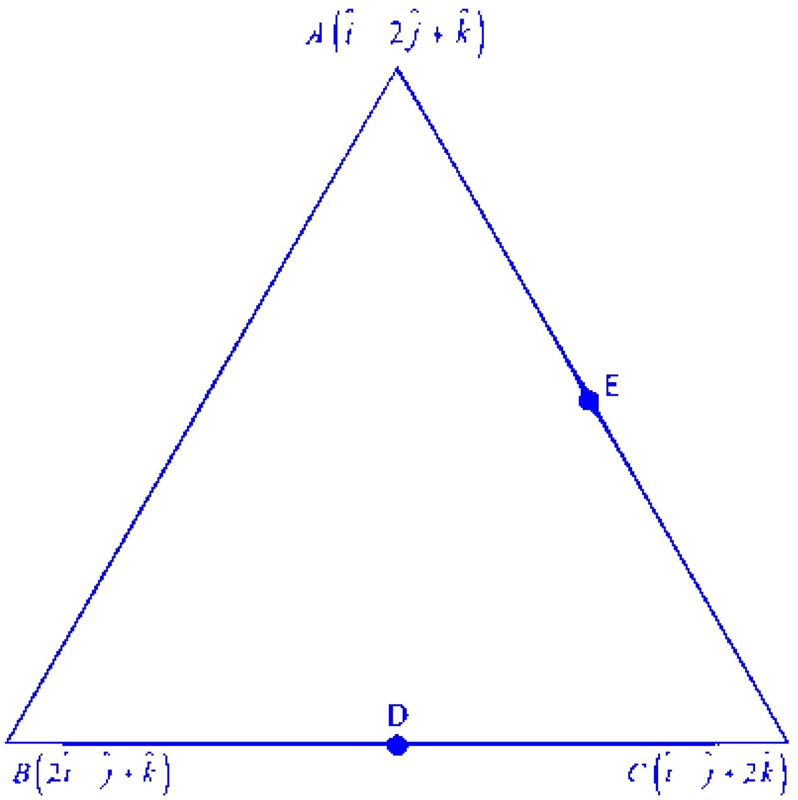
C. $\frac{1}{\sqrt{3}}(\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}})$

D. $\frac{1}{13}(12\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$

Answer: B

Solution:

We have,



$A(\hat{i} - 2\hat{j} + \hat{k})$

$B(2\hat{i} - \hat{j} + \hat{k}) \quad C(\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

Position vector of

$$D = \frac{2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}} + \hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}}}{2}$$

$$= \frac{3}{2}(\hat{\mathbf{i}} - \hat{\mathbf{k}})$$

Position vector of

$$E = \frac{\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}} + \hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}}}{2}$$

$$= \frac{1}{2}(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - \hat{\mathbf{k}})$$

$$\therefore \text{DE} = \frac{1}{2}(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - \hat{\mathbf{k}}) - \frac{3}{2}(\hat{\mathbf{i}} - \hat{\mathbf{k}})$$

$$= -\frac{1}{2}\hat{\mathbf{i}} - \frac{3}{2}\hat{\mathbf{j}} + \hat{\mathbf{k}}$$

$\Rightarrow$ Unit vector along

$$\mathrm{DE}=\frac{\left(-\frac{1}{2}\hat{\mathbf{i}}-\frac{3}{2}\hat{\mathbf{j}}+\hat{\mathbf{k}}\right)}{\sqrt{\left(-\frac{1}{2}\right)^2+\left(\frac{-3}{2}\right)^2+(1)^2}}$$
$$=\frac{1}{\sqrt{14}}(-\hat{\mathbf{i}}-3\hat{\mathbf{j}}+2\hat{\mathbf{k}})$$

Question28

A vector of magnitude $\sqrt{2}$ units along the internal bisector of the angle between the vectors $2\hat{\mathrm{i}}-2\hat{\mathrm{j}}+\hat{\mathrm{k}}$ and $\hat{\mathrm{i}}+2\hat{\mathrm{j}}+2\hat{\mathrm{k}}$ is

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Options:

A. $\hat{\mathbf{j}}+\hat{\mathbf{k}}$

B. $\hat{\mathbf{i}}-\hat{\mathbf{j}}$

C. $\hat{\mathbf{i}}-\hat{\mathbf{k}}$

D. $\hat{\mathbf{i}}+\hat{\mathbf{k}}$

Answer: D

Solution:

Let $\mathbf{a}=2\hat{\mathbf{i}}-2\hat{\mathbf{j}}+\hat{\mathbf{k}}$

$\mathbf{b}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+2\hat{\mathbf{k}}$

$\therefore \quad |a|=3,|b|=3$

A vector along bisectors $=\frac{\mathbf{a}}{|\mathbf{a}|}\pm\frac{b}{|b|}$

$$=\frac{2\hat{\mathbf{i}}-2\hat{\mathbf{j}}+\hat{\mathbf{k}}}{3}\pm\frac{\hat{\mathbf{i}}+2\hat{\mathbf{j}}+2\hat{\mathbf{k}}}{3}$$

$$=\frac{3\hat{\mathbf{i}}+3\hat{\mathbf{k}}}{3},\frac{\hat{\mathbf{i}}-4\hat{\mathbf{j}}-\hat{\mathbf{k}}}{3}$$

$\therefore$ Required vector $=\hat{\mathbf{i}}+\hat{\mathbf{k}}$

$(\because$ Magnitude $=\sqrt{2})$

Question29

If θ is the angle between the vectors $4\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ and $\hat{\mathbf{i}}+3\hat{\mathbf{j}}-2\hat{\mathbf{k}}$, then $\sin 2\theta =$

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Options:

A. $\sqrt{\frac{3}{95}}$

B. $-\sqrt{\frac{3}{95}}$

C. $-\sqrt{\frac{285}{49}}$

D. $\frac{\sqrt{258}}{49}$

Answer: C

Solution:

$$\cos\theta = \frac{(4\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}})\cdot(\hat{\mathbf{i}}+3\hat{\mathbf{j}}-2\hat{\mathbf{k}})}{\sqrt{4^2+(-1)^2+2^2}\cdot\sqrt{1^2+3^2+(-2^2}}$$

$$\cos\theta = \frac{4-3-4}{\sqrt{21}\cdot\sqrt{14}} = \frac{-3}{7\sqrt{6}} \Rightarrow \sin\theta = \sqrt{\frac{285}{294}}$$

$$\text{So, } \sin 2\theta = 2\sin\theta\cos\theta$$

$$= 2\cdot\frac{\sqrt{285}}{7\sqrt{6}}\left(\frac{-3}{7\sqrt{6}}\right) = \frac{-\sqrt{285}}{49}$$

Question30

a, b and c are three vectors such that $|a|=3$, $|b|=2\sqrt{2}$, $|c|=5$ and c is perpendicular to the plane of a and b. If the angle between the vectors a and b is $\frac{\pi}{4}$, then $|\mathbf{a}+\mathbf{b}+\mathbf{c}| =$

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Options:

A. $5 \sqrt{3}$

B. $2 \sqrt{5}$

C. 10

D. $3 \sqrt{6}$

Answer: D

Solution:

We have, $|a|=3,|b|=2 \sqrt{2}$ and $|c|=5$

$\because$ c is perpendicular to the plane of a and b .

$\therefore \quad k(\mathbf{a} \times \mathbf{b}) \Rightarrow \mathbf{c} \perp \mathbf{a}, \mathbf{c} \perp \mathbf{b}$

Now, $|\mathbf{a}+\mathbf{b}+\mathbf{c}|^2=|\mathbf{a}|^2+|b|^2+|c|^2+2$

$(\mathbf{a} \cdot \mathbf{b}+\mathbf{b} \cdot \mathbf{c}+\mathbf{c} \cdot \mathbf{a})$

$=9+8+25+2\left(3 \times 2 \sqrt{2} \cdot \sin \frac{\pi}{4}+0+0\right)$

$|\mathbf{a}+\mathbf{b}+\mathbf{c}|^2=42+12=54$

$\Rightarrow|\mathbf{a}+\mathbf{b}+\mathbf{c}|=3 \sqrt{6}$

Question31

If a, b and c are non-coplanar vectors and the points $\lambda \mathbf{a}+3 \mathbf{b}-\mathbf{c}, \mathbf{a}-\lambda \mathbf{b}+3 \mathbf{c}, 3 \mathbf{a}+4 \mathbf{b}-\lambda \mathbf{c}$ and $\mathbf{a}-6 b+6 \mathbf{c}$ are coplanar, then one of the values of λ is

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Options:

A. 7

B. 5

C. 2

D. 1

Answer: C

Solution:

Let $A(\lambda\mathbf{a}+3\mathbf{b}-\mathbf{c}), B(\mathbf{a}-\lambda\mathbf{b}+3\mathbf{c})$

$\mathbf{c}(3\mathbf{a}+4\mathbf{b}-\lambda\mathbf{c})$ and $D(\mathbf{a}-6\mathbf{b}+6\mathbf{c})$

$\mathbf{AD}=(1-\lambda)\mathbf{a}-9\mathbf{b}+7\mathbf{c}$

$\mathbf{BD}=(\lambda-6)\mathbf{b}+3\mathbf{c}$

$\mathbf{CD}=-2\mathbf{a}-10\mathbf{b}+(6+\lambda)\mathbf{c}$

$\because A, B, C$ and D are coplanar

$$\Rightarrow\begin{vmatrix}1-\lambda & -9 & 7\\ 0 & \lambda-6 & 3\\ -2 & -10 & 6+\lambda\end{vmatrix}=0$$

$\Rightarrow\quad(1-\lambda)\left[\lambda^2-36+30\right]$

$-2[-27-7\lambda+42]=0$

$\Rightarrow\quad(1-\lambda)\left(\lambda^2-6\right)-2(15-7\lambda)=0$

$\Rightarrow\quad\lambda^2-6-\lambda^3+6\lambda-30+14\lambda=0$

$\Rightarrow\quad-\lambda^3+\lambda^2+20\lambda-36=0$

$\Rightarrow\quad\lambda^3-\lambda^2-20\lambda+36=0$

$\Rightarrow\quad(\lambda-2)\left(\lambda^2+\lambda-18\right)=0\Rightarrow\lambda=2$

Question32

If two vectors a and b which are perpendicular to each other are such that $|\mathbf{a}|=8$ and $|\mathbf{b}|=3$, then $|\mathbf{a}-2b|=$

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Options:

A. 10

B. 2

C. 6

D. 12

Answer: A

Solution:

Given, $\mathbf{a} \cdot \mathbf{b} = 0$ and $|\mathbf{a}| = 8, |\mathbf{b}| = 3$

$Now, |\mathbf{a} - 2\mathbf{b}|^2 = (\mathbf{a} - 2\mathbf{b}) \cdot (\mathbf{a} - 2\mathbf{b})$

$= |\mathbf{a}|^2 + 4|\mathbf{b}|^2$

$(\because \mathbf{a} \cdot \mathbf{b} = 0)$

$= (8)^2 + 4(3)^2$

$= 64 + 4 \times 9 = 64 + 36 = 100$

$\therefore \quad |a - 2b| = \sqrt{100} = 10$

Question33

Let a and b be non-collinear vectors. If the vectors $(\lambda - 1)\mathbf{a} + 2\mathbf{b}$ and $3\mathbf{a} + \lambda\mathbf{b}$ are collinear, then the set of all possible values of λ is

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Options:

A. $\{2, 3\}$

B. $\{-2, 3\}$

C. $\{-2, -3\}$

D. $\{2, -3\}$

Answer: B

Solution:

Given, $\mathbf{a}$ and $\mathbf{b}$ be non-collinear vectors.

If the vectors $(\lambda - 1)\mathbf{a} + 2\mathbf{b}$ and $3\mathbf{a} + \lambda\mathbf{b}$ are collinear.

Now, $(\lambda - 1)\mathbf{a} + 2\mathbf{b} = P(3\mathbf{a} + \lambda\mathbf{b})$

$\Rightarrow \{(\lambda - 1) - 3P\}\mathbf{a} + \{2 - \lambda P\}\mathbf{b} = 0$

Since, $\mathbf{a}$ and $\mathbf{b}$ are non-collinear,

So, $\quad \lambda - 1 - 3P = 0$ and $2 - \lambda P = 0$

$$\Rightarrow \quad \lambda - 1 = 3 \cdot P \text{ and } P = \frac{2}{\lambda}$$
$$\Rightarrow \quad \lambda - 1 \Rightarrow \frac{6}{\lambda} = \lambda^2 - \lambda - 6 = 0$$
$$\Rightarrow \quad \lambda = -2 \text{ or } 3$$

Thus, the set of all possible value of λ is $\{-2, 3\}$.

Question34

Vectors $\mathbf{p} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}, \mathbf{q} = d\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ and $\mathbf{r} = 3\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ forming a $\triangle ABC$ are such that $\mathbf{p} = \mathbf{q} + \mathbf{r}$. If the area of $\triangle ABC$ is $5\sqrt{6}$ sq. units, then the sum of the absolute values of a, b, c is

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Options:

A. 14

B. 13

C. 12

D. 10

Answer: A

Solution:

Vectors, $\mathbf{p}=a\hat{\mathbf{i}}+b\hat{\mathbf{j}}+c\hat{\mathbf{k}}$, $\mathbf{q}=d\hat{\mathbf{i}}+3\hat{\mathbf{j}}+4\hat{\mathbf{k}}, \mathbf{r}=3\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}}$ forming a $\triangle ABC$ are such that, $\mathbf{p}=\mathbf{q}+\mathbf{r}$.

where, $a\hat{\mathbf{i}}+b\hat{\mathbf{j}}+c\hat{\mathbf{k}}$

$$=(d\hat{\mathbf{i}}+3\hat{\mathbf{j}}+4\hat{\mathbf{k}})+(3\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}})$$
$$\Rightarrow a\hat{\mathbf{i}}+b\hat{\mathbf{j}}+c\hat{\mathbf{k}}=(d+3)\hat{\mathbf{i}}+4\hat{\mathbf{j}}+2\hat{\mathbf{k}}$$

Comparing coefficient of $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$, we get

$a=d+3, b=4$ and $c=2 \qquad \ldots(i)$

Now, area of $\triangle ABC$ is $5\sqrt{6}$ sq units

$$\Rightarrow \quad \frac{1}{2}|\mathbf{q}\times\mathbf{r}|=5\sqrt{6}$$

$$\Rightarrow \frac{1}{2}\left\|\begin{array}{ccc}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ d & 3 & 4 \\ 3 & 1 & -2\end{array}\right\|=5\sqrt{6}$$
$$\Rightarrow \frac{1}{2}\sqrt{(-10)^2+(2d+12)^2+(d-9)^2}=5\sqrt{6}$$
$$\Rightarrow \quad \frac{1}{4}\left(5d^2+30d+325\right)=150$$
$$\Rightarrow \quad 5d^2+30d+325=600$$
$$\Rightarrow \quad d^2+6d-55=0$$
$$\therefore d=-11,5 \quad a=-8,8 \quad \text{[from Eq. (i)]}$$
$$b=4, \text{ and } c=2 \quad |a|+|b|+|c| \quad \text{Now,}$$
$$=|\pm 8|+|4|+|2|=14$$

Question35

b and c are non-collinear vectors and $(\mathbf{c}\cdot\mathbf{c})\mathbf{a}=\mathbf{c}$. If $(\mathbf{a}\cdot\mathbf{c})\mathbf{b}-(\mathbf{a}\cdot\mathbf{b})\mathbf{c}+(\mathbf{a}\cdot\mathbf{b})\mathbf{b}=(4-2\beta-\sin\alpha)\mathbf{b}+\left(\beta^2-1\right)\mathbf{c}$, then $\sin(\alpha+\beta)=$

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Options:

A. 0

B. 1

C. $\sin 1$

D. $\cos 1$

Answer: B

Solution:

we have

$$(\mathbf{a} \cdot \mathbf{c})\mathbf{b}-(\mathbf{a} \cdot \mathbf{b})\mathbf{c}+(\mathbf{a} \cdot \mathbf{b})\mathbf{b}$$
$$=(4-2\beta-\sin\alpha)\mathbf{b}+\left(\beta^2-1\right)\mathbf{c}$$

Comparing on both side, we get

$$\mathbf{a} \cdot \mathbf{c}+\mathbf{a} \cdot \mathbf{b}=4-2\beta-\sin\alpha \quad \ldots(i)$$
$$\text{and} \quad -\mathbf{a} \cdot \mathbf{b}=\left(\beta^2-1\right) \quad \ldots(ii)$$

Given, $(\mathbf{c} \cdot \mathbf{c})\mathbf{a}=\mathbf{c}$

$$\Rightarrow \quad (\mathbf{c} \cdot \mathbf{c})(\mathbf{a} \cdot \mathbf{c})=(\mathbf{c} \cdot \mathbf{c})$$
$$\Rightarrow \quad \mathbf{a} \cdot \mathbf{c}=1$$

Now, from Eq. (i),

$$1+\mathbf{a} \cdot \mathbf{b}=4-2\beta-\sin\alpha \quad \ldots(iii)$$

Using Eq. (ii), we get

$$1+1-\beta^2=4-2\beta-\sin\alpha$$
$$\Rightarrow \beta^2-2\beta+2=\sin\alpha$$
$$\Rightarrow (\beta-1)^2+1=\sin\alpha$$

It is possible only when $\beta=1$ and $\sin\alpha=1$

$$\Rightarrow \quad \beta=1 \text{ and } \alpha=\frac{\pi}{2}$$

Now, $\sin(\alpha+\beta)=\sin\left(\frac{\pi}{2}+1\right) \equiv \cos(1)$

Question36

If the position vectors of P and Q are $\hat{\mathbf{i}}+2\hat{\mathbf{j}}-7\hat{\mathbf{k}}$ and $5\hat{\mathbf{i}}-3\hat{\mathbf{j}}+4\hat{\mathbf{k}}$ respectively, then the cosine of the angle between PQ and Z-axis is

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Options:

A. $\frac{4}{\sqrt{162}}$

B. $\frac{11}{\sqrt{162}}$

C. $\frac{5}{\sqrt{162}}$

D. $\frac{-5}{\sqrt{162}}$

Answer: B

Solution:

Given the position vectors of $\mathbf{P}$ and $\mathbf{Q}$ are:

$$\mathbf{P} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 7\hat{\mathbf{k}}$$
$$\mathbf{Q} = 5\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$$

To find the vector $\mathbf{PQ}$, we calculate:

$$\mathbf{PQ} = \mathbf{Q} - \mathbf{P} = (5\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) - (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 7\hat{\mathbf{k}})$$

Simplifying the components, we get:

$$\mathbf{PQ} = (5-1)\hat{\mathbf{i}} + (-3-2)\hat{\mathbf{j}} + (4+7)\hat{\mathbf{k}} = 4\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 11\hat{\mathbf{k}}$$

The direction ratios of $\mathbf{PQ}$ are $a = 4$, $b = -5$, and $c = 11$.

To find the cosine of the angle γ between the vector $\mathbf{PQ}$ and the Z-axis, we use the relation:

$$\cos\gamma = \frac{c}{\sqrt{a^2+b^2+c^2}}$$

Substituting the values, we have:

$$\cos\gamma = \frac{11}{\sqrt{4^2+(-5)^2+11^2}} = \frac{11}{\sqrt{16+25+121}} = \frac{11}{\sqrt{162}}$$

Question37

a, b, c are three-unit yectors such that $|\mathbf{a}+\mathbf{b}+\mathbf{c}| = 1$ and a is perpendicular to b. If c makes angles α, β with a, b respectively, then $\cos\alpha + \cos\beta =$

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Options:

A. 1

B. -1

C. 2

D. -2

Answer: B

Solution:

To solve this problem, we need to evaluate the given vector conditions.

We are given that $|\mathbf{a}+\mathbf{b}+\mathbf{c}|=1$, where $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are unit vectors, meaning that $|\mathbf{a}|^2=|\mathbf{b}|^2=|\mathbf{c}|^2=1$. Additionally, $\mathbf{a}$ is perpendicular to $\mathbf{b}$, which implies $\mathbf{a} \cdot \mathbf{b}=0$.

To proceed, we expand and square the magnitude of the vector sum:

$|\mathbf{a}+\mathbf{b}+\mathbf{c}|^2=1$

$|\mathbf{a}|^2+|\mathbf{b}|^2+|\mathbf{c}|^2+2(\mathbf{a} \cdot \mathbf{b})+2(\mathbf{b} \cdot \mathbf{c})+2(\mathbf{c} \cdot \mathbf{a})=1$

Substituting the known values:

$1+1+1+0+2(\mathbf{b} \cdot \mathbf{c})+2(\mathbf{c} \cdot \mathbf{a})=1$

This simplifies to:

$3+2 \cos \beta+2 \cos \alpha=1$

Therefore:

$2 \cos \alpha+2 \cos \beta=-2$

Dividing the entire equation by 2 gives:

$\cos \alpha+\cos \beta=-1$

Question38

If a is a vector such that $\mathbf{a} \times \hat{\mathbf{i}}=\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $\mathbf{a} \cdot \hat{\mathbf{i}}=1$, then equation of the line passing through the point $\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and parallel to a is

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Options:

A. $\mathbf{r}=(t+1) \hat{i}+(1-t) \hat{j}+(t+1) \hat{k}$

B. $r=(t+1) \hat{\mathbf{i}}-(2 t-1) \hat{\mathbf{j}}+t \hat{\mathbf{k}}$

C. $\mathbf{r}=\hat{\mathbf{i}}+t \hat{\mathbf{j}}-t \hat{\mathbf{k}} \mathrm{c}$

D. $\mathbf{r} = 5t\hat{\mathbf{i}} + 7t\hat{\mathbf{j}} + \hat{\mathbf{k}}$

Answer: A

Solution:

To determine the equation of the line through a given point and parallel to a vector $\mathbf{a}$, consider the following:

Given:

$\mathbf{a} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} + \hat{\mathbf{k}}$

$\mathbf{a} \cdot \hat{\mathbf{i}} = 1$

First, find the vector $\mathbf{a}$. Assume $\mathbf{a}$ can be expressed as $x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$. We have:

$(x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}) \times \hat{\mathbf{i}} = \hat{\mathbf{j}} + \hat{\mathbf{k}}$

Breaking down the cross product:

$x\hat{\mathbf{i}} \times \hat{\mathbf{i}} = 0$

$y\hat{\mathbf{j}} \times \hat{\mathbf{i}} = -y\hat{\mathbf{k}}$

$z\hat{\mathbf{k}} \times \hat{\mathbf{i}} = z\hat{\mathbf{j}}$

Combining, we get:

$-y\hat{\mathbf{k}} + z\hat{\mathbf{j}} = \hat{\mathbf{j}} + \hat{\mathbf{k}}$

Equating components:

$z = 1$

$-y = 1 \Rightarrow y = -1$

Given $\mathbf{a} \cdot \hat{\mathbf{i}} = 1$:

$(x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}) \cdot \hat{\mathbf{i}} = x = 1$

Thus, we find:

$\mathbf{a} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$

Now, the equation of the line passing through the point $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and parallel to the vector $\mathbf{a}$ is:

$\mathbf{r} = (\text{Passing through point}) + t(\text{Parallel vector})$

Substitute:

$\mathbf{r} = (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) + t(\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})$

Simplify:

$\mathbf{r} = (1+t)\hat{\mathbf{i}} + (1-t)\hat{\mathbf{j}} + (1+t)\hat{\mathbf{k}}$

This is the equation of the line.

Question39

The position vectors of the point A, B are a, b respectively. If the position vector of the point C is $\frac{a}{2}+\frac{b}{3}$, then

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Options:

A. C lies inside $\triangle OAB$

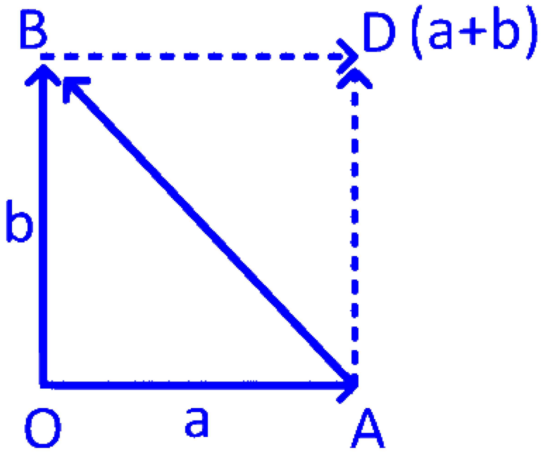
B. C lies outside $\triangle OAB$ but inside $\angle AOB$

C. C lies outside $\triangle OAB$ but inside $\angle OAB$

D. C lies outside $\triangle OAB$ but inside $\angle OBA$

Answer: A

Solution:



Mid-point of AB is $\frac{\mathbf{a}}{2}+\frac{\mathbf{b}}{2}$ and position vector of C is $\frac{\mathbf{a}}{2}+\frac{\mathbf{b}}{3} \Rightarrow C$ lies inside $\triangle OAB$.

Question40

If $|\mathbf{a}|=1, |\mathbf{b}|=2, |\mathbf{a}-\mathbf{b}|^2+|\mathbf{a}+2\mathbf{b}|^2=20$, then $(a, b)=$

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Options:

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

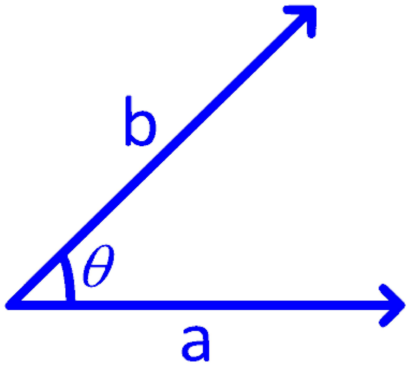
C. $\frac{\pi}{6}$

D. $\frac{2\pi}{3}$

Answer: D

Solution:

$$|\mathbf{a}-\mathbf{b}|^2+|\mathbf{a}+2\mathbf{b}|^2=20 \quad \text{[given]}$$
$$\Rightarrow |a|^2+|b|^2-2a\cdot b+|a|^2+4|b|^2$$
$$+4a\cdot b=20$$
$$\Rightarrow 2|a|^2+5|b|^2+2a\cdot b=20$$



$$|\mathbf{a}|=1, |\mathbf{b}|=2$$
$$\mathbf{a}\cdot\mathbf{b}=|\mathbf{a}||\mathbf{b}|\cos\theta$$
$$=2\cos\theta$$
$$2+20+4\cos\theta=20$$
$$\Rightarrow \quad \cos\theta=-\frac{1}{2}$$
$$\Rightarrow \quad \theta=\frac{2\pi}{3}$$
